Weight of Fitness Deviation Governs Strict Physical Chaos in Evolutionary Game Dynamics

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Replicator equation - a paradigm equation in evolutionary game dynamics - mathematizes the frequency dependent selection of competing strategies vying to enhance their fitness (quantified by the average payoffs) with respect to the average fitnesses of the evolving population under consideration. Here, we deal with two discrete versions of the replicator equation employed to study evolution in a population where any two players' interaction is modeled by a two-strategy symmetric normal-form game. There are twelve distinct classes of such games, each typified by a particular ordinal relationship among the elements of the corresponding payoff matrix. Here we find the sufficient conditions for the existence of asymptotic solutions of the replicator equations such that the solutions—fixed points, periodic orbits, and chaotic trajectories—are all strictly physical, meaning that the frequency of any strategy lies inside the closed interval zero to one at all times. Thus, we elaborate which of the twelve types of games are capable of showing meaningful physical solutions and for which of the two types of replicator equation. Subsequently, we introduce the concept of the weight of fitness deviation that is the scaling factor in a positive affine transformation connecting two payoff matrices such that the corresponding one-shot games have exactly same Nash equilibria and evolutionary stable states. The weight also quantifies how much the excess of fitness of a strategy over the average fitness of the population affects the per capita change in the frequency of the strategy. Intriguingly, the weight's variation is capable of making the Nash equilibria and the evolutionary stable states useless by introducing strict physical chaos in the replicator dynamics based on the normal-form game. Additionally, we also see the effect of mutation on the aforementioned results.